FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2009

APPLIED MATH, PAPER-I

S.No.	
R.No.	

TIME ALLOWED: 3 HOURS **MAXIMUM MARKS:100**

NOTE:	**(i)** Attempt **FIVE** question in all by selecting at least **TWO** questions from **SECTION – A** and **THREE** question from **SECTION – B**. All questions carry **EQUAL** marks. **(ii) Use of Scientific Calculator is allowed.**

SECTION – A

Q.1. (a) Show that in orthogonal coordinates: **(5+5)**

(i) $$\nabla \times (A_1 e_1) = \frac{e_2}{h_1 h_3}\frac{\partial}{\partial u_3}(A_1 h_1) - \frac{e_3}{h_1 h_2}\frac{\partial}{\partial u_2}(A_1 h_1),$$

(ii) $$\nabla \bullet (A_1 e_1) = \frac{1}{h_1 h_2 h_3}\frac{\partial}{\partial u_1}(A_1 h_2 h_3).$$

(b) Write Laplace's equation in parabolic cylindrical coordinates. **(10)**

Q.2. (a) Evaluate $\iint_S \vec{A} \bullet \hat{n}\, ds$, where $\vec{A} = z\hat{i} + x\hat{j} - 3y^2 \hat{k}$ and s is the surface of cylinder $x^2+y^2=16$ included in the first octant between $z=0$ and $z=5$. **(10)**

(b) Verify Green's theorem in the plane for

$$\oint_c (xy + y^2)dx + x^2 dy$$

where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

Q.3. (a) Find centre of mass of a right circular solid cone of height *h*. **(10)**

(b) A light thin rod, 4m long, can turn in a vertical plane about one of its point which is attached to a pivot. If weights of *3kg and 4kg* are suspended from its ends it rests in a horizontal position. Find the position of the pivot and its reaction on the rod. **(10)**

SECTION – B

Q.4. (a) Find the radial and transverse components of the velocity of a particle moving along the curve $ax^2+by^2=1$ at any time t if the polar angle $\theta = ct^2$. **(10)**

(b) A particle is projected vertically upwards. After a time t, another particle is sent up from the same point with the same velocity and meets the first a height h during the downward flight of the first. Find the velocity of projection. **(10)**

Q.5. (a) If a point P moves with a velocity v given by

$$v^2=n^2 (ax^2+bx+c),$$

show that P executes a simple harmonic motion. Find also, the centre, the amplitude and the time period of the motion. **(10)**

(b) A particle of mass m moves on xy-plane under the force **(10)**

$$\vec{F} = -\frac{k}{r^4}\vec{r},$$

where r is its distance from the origin O. If it starts on the positive x-axis at a distance "a" from O with speed v_o in a direction making an angle θ with the positive x-direction, prove that at time t,

$$\ddot{r} = \frac{ma^2 v_o^2 \sin^2\theta - k}{mr^3}$$

Q.6. (a) Define angular momentum and prove that rate of change of angular momentum of a particle about a point O is equal to the tarque (about O) of the force acting on the particle. **(10)**

(b) Find the least speed with which a particle must be projected so that it passes through two points P and Q at heights h_1 and h_2, respectively. **(10)**

Q.7. (a) Discuss the polar form of an orbit and prove that when a particle moves under central force, the areal velocity is constant. **(10)**

(b) A particle moves under a central repulsive force μ/r^3 and is projected from an apse at a distance "a" with velocity V. Show that the equation to the path is **(10)**

$$r \cos p\,\theta = a$$

and that the angle θ described in time t is

$$\frac{1}{p}\tan^{-1}\frac{pVt}{a},$$

where $$p^2 = 1 + \frac{\mu}{a^2 v^2}, \qquad \mu = GM.$$

Q.8. (a) Define the terms moment of inertia and product of inertia, and find the moment of inertia of uniform solid sphere of mass m and radius "a". **(10)**

(b) Let AB and BC be two equal similar rods freely hinged at B and lie in a straight line on the smooth table. The end A is struck by a blow P perpendicular to AB. Show that the resulting velocity of A is $3\frac{1}{2}$ times that of B. **(10)**

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APPLIED MATH, PAPER-II

S.No.	
R.No.	

TIME ALLOWED: 3 HOURS **MAXIMUM MARKS:100**

NOTE:	(i) Attempt **FIVE** question in all by selecting at least **TWO** questions from **SECTION–A, ONE** question from **SECTION–B** and **TWO** questions from **SECTION–C**. All questions carry **EQUAL** marks. **(ii) Use of Scientific Calculator is allowed.**

SECTION – A

Q.1. (a) Using method of variation of parameters, find the general solution of the differential equation.

$$y'' - 2y' + y = \frac{e^x}{x} \quad . \qquad \textbf{(10)}$$

(b) Find the recurrence formula for the power series solution around $x = 0$ for the differential equation

$$y'' + xy = e^{x+1}. \qquad \textbf{(10)}$$

Q.2. (a) Find the solution of the problem **(10)**

$$u'' + 6u' + 9u = 0$$
$$u(0) = 2, \quad u'(0) = 0$$

(b) Find the integral curve of the equation

$$xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} = -(x^2 + y^2) \, . \qquad \textbf{(10)}$$

Q.3. (a) Using method of separation of variables, solve **(10)**

$$\frac{\partial^2 u}{\partial t^2} = 900\frac{\partial^2 u}{\partial x^2} \qquad \begin{cases} 0 < x < 2 \\ t > 0 \end{cases} \quad ,$$

subject to the conditions

$$u(0,t) = u(2,t) = 0$$
$$u(x,0) = 0 \qquad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 30\, sin\, 4\pi\, x.$$

(b) Find the solution of **(10)**

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 4e^{3y} + \cos x \, .$$

SECTION – B

Q.4. (a) Define alternating symbol $\in_{ijk}$ and Kronecker delta δ_{ij}. Also prove that **(10)**

$$\in_{ijk} \in_{lmk} = \delta_{il}\, \delta_{jm} - \delta_{im}\, \delta_{jl} \; .$$

(b) Using the tensor notation, prove that

$$\nabla \times (\vec{A} \times \vec{B}) = \vec{A}(\nabla \bullet \vec{B}) - \vec{B}(\nabla \bullet \vec{A}) + (\vec{B} \bullet \nabla)\vec{A} - (\vec{A} \bullet \nabla)\vec{B} \qquad \textbf{(10)}$$

Q.5. (a) Show that the transformation matrix

$$\mathbf{T} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

is orthogonal and right-handed. **(10)**

(b) Prove that **(10)**

$$l_{ik}\ l_{jk} = \delta_{ij}$$

where l_{ik} is the cosine of the angle between *ith-axis* of the system K' and *jth-axis* of the system *K*.

SECTION – C

Q.6. (a) Use Newton's method to find the solution accurate to within 10^{-4} for the equation **(10)**

$$x^3 - 2x^2 - 5 = 0, \quad [1, 4].$$

(b) Solve the following system of equations, using Gauss-Siedal iteration method **(10)**

$4x_1 - x_2 + x_3 = 8,$
$2x_1 + 5x_2 + 2x_3 = 3,$
$x_1 + 2x_2 + 4x_3 = 11.$

Q.7. (a) Approximate the following integral, using Simpson's $\frac{1}{3}$ rules **(10)**

$$\int_0^1 x^2 e^{-x}\,dx.$$

(b) Approximate the following integral, using Trapezoidal rule **(10)**

$$\int_0^{\pi/4} e^{3x} \sin 2x\,dx.$$

Q.8. (a) The polynomial **(10)**

$f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

has one real zero in [-1, 0]. Attempt approximate this zero to within 10^{-6}, using the Regula Falsi method.

(b) Using Lagrange interpolation, approximate. **(10)**

f(1.15), if f(1) = 1.684370, f(1.1) = 1.949477, f(1.2) = 2.199796, f(1.3) = 2.439189, f(1.4) = 2.670324

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