SECTION - A

Q.1. (a) Find the divergence and curl \( \mathbf{f} \) if \( \mathbf{f} = 2xyz \mathbf{\hat{i}} + (x^2z + 2y) \mathbf{\hat{j}} + (x^2y + 3z^2) \mathbf{\hat{k}} \) (10)

(b) Also find a function \( \phi \) such that \( \nabla \phi = \mathbf{f} \) (10)

Q.2. (a) Find the volume \( \iint_{R} xy \, dA \) where \( R \) is the region bounded by the line \( y = x - 1 \) and the parabola \( y^2 = 2x + 6 \) (10)

(b) Evaluate the following line integral:
\[
\int_{c} y^2 \, dx + x \, dy \quad \text{where} \quad c = c_2 \text{ is the line segment joining the points (-5, -3) to (0, 2), and } c = c_4 \text{ is the arc of the parabola } x = 4 - y^2 .
\]

Q.3. (a) Three forces P, Q and R act at a point parallel to the sides of a triangle ABC taken in the same order. Show that the magnitude of the resultant is
\[
\sqrt{p^2 + q^2 + r^2 - 2QR \cos A - 2RP \cos B - 2PQ \cos C}
\]

(b) A hemispherical shell rests on a rough inclined plane whose angle of friction is \( \lambda \). Show that the inclination of the plane base to the horizontal cannot be greater than \( \arcsin(2 \sin \lambda) \) (10)

Q.4. (a) A uniform square lamina of side \( 2a \) rests in a vertical plane with two of its sides in contact with two smooth pegs distant \( b \) apart and in the same horizontal line. Show that if \( \frac{\theta}{\sqrt{2}} < b < a \), a non symmetric position of equilibrium is possible in which \( b(\sin \theta + \cos \theta) = a \) (10)

(b) Find the centre of mass of a semi circular lamina of radius \( a \) whose density varies as the square of the distance from the centre. (10)
Q.5. (a) Evaluate the integral \( \int_0^1 \int_{2x}^{2y} (x^2 + y^2) \, dy \, dx \) also show that the order of integration is immaterial.

(b) Find the directional derivative of the function at the point P along z – axis
\( f(x, y) = 4xz^3 - 3x^2y^2z, \quad P = (2, -1, 2) \)

SECTION – B

Q.6. (a) A particle is moving along the parabola \( x^2 = 4ay \) with constant speed \( v \). Determine the tangential and the normal components of its acceleration when it reaches the point whose abscissa is \( \sqrt{5a} \)

(b) Find the distance travelled and the velocity attained by a particle moving in a straight line at any time \( t \), if it starts from rest at \( t = 0 \) and is subject to an acceleration \( t^2 + \sin t + e^t \)

Q.7. (a) A particle moves in the xy – plane under the influence of a force field which is parallel to the axis of y and varies as the distance from \( x \) – axis. Show that, if the force is repulsive, the path of the particle supposed not straight and then

\[ y = a \cosh nx + a \sinh nx \]

where \( a \) and \( b \) are constants.

(b) Discuss the motion of a particle moving in a straight line with an acceleration \( x^3 \), where \( x \) is the distance of the particle from a fixed point O on the line, if it starts at \( t = 0 \) from a point \( x = c \) with the velocity \( c^2 / \sqrt{2} \).

Q.8. (a) A battleship is steaming ahead with speed \( V \) and a gun is mounted on the battleship so as to point straight backwards and is set at angle of elevation \( \alpha \). If \( v_0 \) is the speed of projection (relative to the gun) show that the range is \( \frac{2v_0 \sin \alpha (v_0 \cos \alpha - V)}{g} \)

(b) Show that the law of force towards the pole of a particle describing the survey \( r^n = a^n \cos n \theta \) is given by \( f = \frac{(n + 1)h^2 a^{2n}}{r^{2n+3}} \) where \( h \) is a constant.
FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2011

APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS       MAXIMUM MARKS: 100

NOTE: (i) Attempt FIVE questions in all by selecting THREE questions from SECTION – A and TWO questions from SECTION – B. All questions carry equal marks.
(ii) Use of Scientific Calculator is allowed.
(iii) Extra attempt of any question or any part of the attempted question will not be considered.

SECTION - A

Q.1. (a) Solve by method of variation of parameter
\[ \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = xe^x \ln x \]  

(b) Solve first order non-linear differential equation
\[ x \frac{dy}{dx} + y = y^2 \ln x \]  

Q.2. (a) Solve
\[ c^2 u_{xx} = u_{tt} \]
\[ u(0,t) = 0 \]
\[ u(l,t) = 0 \]
\[ u(x,0) = \lambda \sin \left( \frac{\pi}{l} x \right) \]
\[ u_t(x,0) = 0 \]  

(b) Solve
\[ x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y) z \]

Q.3. (a) Work out the two dimensional metric tensor for the coordinates p and q given by
\[ p = (xy) \frac{1}{3}, \quad q = (x^2 / y)^{\frac{1}{3}} \]  

(b) Prove that
\[ r_{ab}^d = \frac{1}{2} g^{dc} \left( g_{ac,b} + g_{bc,a} - g_{ab,c} \right) \]
Q.4. (a) Work out the Christoffel symbols for the following metric tensor

\[ g_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix} \]

(b) Work out the covariant derivative of the tensor with components

\[
\begin{pmatrix}
 r \cos \theta & a r \sin \varphi & a \\
 \sin \theta \sin \varphi & a \sin \vartheta \cos \varphi & a \\
 \cos \varphi & a \sin \varphi & 0
\end{pmatrix}
\]

Q.5. (a) Find recurrence relations and power series solution of \((x - 3)y' + 2y = 0\) 
(b) Solve the Cauchy Euler’s equation \(x^4 y''' + 2x^3 y'' - x^2 y' + xy = 1\)

SECTION – B

Q.6. (a) Find the positive solution of the following equation by Newton Raphson method

\[ 2 \sin x = x \]

(b) Solve the following system by Jacobi method:

\[
\begin{align*}
10x_1 - 8x_2 &= -6 \\
-8x_1 + 10x_2 - x_3 &= 9 \\
-x_2 + 10x_3 &= 28
\end{align*}
\]

Q.7. (a) Evaluate the following by using the trapezoidal rule.

\[ \int_0^1 (x+1) \, dx \]

(b) Evaluate the following integral by using Simpson’s rule

\[ \int_0^1 e^x \, dx \]

Q.8. (a) Solve the following equation by regular falsi method:

\[ 2x^3 + x - 2 = 0 \]

(b) Calculate the Lagrange interpolating polynomial using the following table:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

also calculate \( f(0.5) \).

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