FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2013

Roll Number

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE:
- **(i)** Candidate must write **Q.No.** in the **Answer Book** in accordance with **Q.No.** in the **Q.Paper**.
- **(ii)** Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B.** All questions carry **EQUAL** marks.
- **(iii)** **Use of Calculator is allowed.**
- **(iv)** Extra attempt of any question or any part of the attempted question will not be considered.

SECTION-A

Q.1. **(a)** Find a function φ such that $\nabla\varphi = \overleftrightarrow{f}$ **(10)**

$$\overleftrightarrow{f} = x\hat{i} + 2y\hat{j} + 2\hat{k}$$

(b) Prove that **(10)**

$$\nabla\varphi^n = n\varphi^{n-1}\nabla\varphi$$

Q.2. **(a)** Show that for any vectors $\vec{a}$ and $\vec{b}$ **(10)**

$$\left|\vec{a}+\vec{b}\right|^2 + \left|\vec{a}-\vec{b}\right|^2 = 2\left(\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2\right)$$

(b) Prove that **(10)**

$$\left(\vec{a}\times\vec{b}\right)\bullet\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right) = \left(\vec{a}\bullet\vec{b}\times\vec{c}\right)^2$$

Q.3. **(a)** The greatest result that two forces can have is of magnitude P and the least is of magnitude Q. Show That when they act an angle α their resultant is of magnitude **(10)**

$$\sqrt{P^2\cos^2\alpha/2 + Q^2\sin^2\alpha/2}$$

(b) A uniform rod of length $2a$ rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall. Show that in the position of equilibrium the rod is inclined to the wall at an angle $\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$ **(10)**

Q.4. **(a)** Three forces P, Q and R act along the BC, CA and AB respectively of triangle ABC. Prove that if $P\cos A + Q\cos B + R\cos C = 0$, then the line of action of the resultant passes through the circum center of the triangle. **(10)**

(b) A sphere of weight W and radius a is suspended by a string of length l from a point P and a weight w is also suspended from P by a string sufficiently long for the weight to hang below the sphere. Show that the inclination of the first string to the vertical is **(10)**

$$\sin^{-1}\left(\frac{wa}{(W+w)(a+l)}\right)$$

Q.5. **(a)** Find the volume $\iint_R \left(x^3 + 4y\right)\, dA$ where R is the region bounded by the parabola $y = x^2$ and the line $y = 2x$. **(10)**

(b) Evaluate the following line integral **(10)**

$$\int_c x^2\, dy$$

bonded by the triangle having the vertices (-1,0) to (2,0), and (1,1)

SECTION-B

Q.6. **(a)** The position of a particle moving along an ellipse is given by $\overset{\leftrightarrow}{r} = a\cos t\,\hat{i} + b\sin t\hat{j}$. If $a > b$, find the position of the particle where its velocity has maximum or minimum magnitude. **(10)**

(b) Prove that the speed at any point of a central orbit is given by: **(10)**

$$vp = h,$$

When h is the areal speed and p is the perpendicular distance from the centre of force, of the tangent at the point, Find the expression for v when a particle subject to the inverse square law of force describes an ellipse, a parabolic and hyperbolic orbit.

Q.7. **(a)** A particle is moving with the uniform speed v along the curve **(10)**

$$x^2 y = a\left(x^2 + \frac{a^2}{\sqrt{5}}\right)$$

Show that its acceleration has the maximum value at $\dfrac{10v^2}{9a}$

(b) An aeroplane is flying with uniform speed v_0 in an arc of a vertical circle of radius a, whose centre is a height h vertically above a point O of the ground. If a bomb is dropped from the aeroplane when at a height Y and strikes the ground at O, show that Y satisfies the equations **(10)**

$$KY^2 + Y\left(a^2 - 2hK\right) + K\left(h^2 - a^2\right) = 0,$$

where $K = h + \dfrac{ga^2}{2v^2{}_0}$

Q.8. **(a)** Find the tangential and normal components of the acceleration of a particle describing the ellipse **(10)**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

With uniform speed v when the particle is a $a > b$

(b) Find the velocity acquired by a block of wood of mass M lb., which is free to recoil when it is struck by a bullet of mass m lb. moving with velocity v in a direction passing through the centre of gravity. If the bullet is embedded a ft., show that the resistance of the wood to the bullet, supposed uniform, is $\dfrac{Mm^2}{2(M+m)ga}$ lb.wt. and that the time of penetration is $\dfrac{2a}{v}$ sec., during which time the block will move $\dfrac{ma}{m+M}$ ft. **(10)**

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Roll Number

APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE:
- **(i)** Candidate must write **Q. No.** in the **Answer Book** in accordance with **Q. No.** in the **Q. Paper.**
- **(ii)** Attempt **FIVE** questions in all by selecting **TWO** questions from **SECTION-A** and **ONE** question from **SECTION-B** and **TWO** questions from **SECTION-C ALL** questions carry **EQUAL** marks**.**
- **(iii)** Extra attempt of any question or any part of the attempted question will not be considered.
- **(iv)** **Use of Calculator is allowed.**

SECTION-A

Q.No.1. Solve the following equations:

(a) $\frac{d^3y}{dx^3}+\frac{dy}{dx}=Sec^2x$ **(10)**

(b) $\frac{2dy}{dx}-\frac{x}{y}+x^3Cos\ y=0$ **(10)**

Q.No.2. **(a)** Find the power series solution of the differential equation $(1-x^2)y^{//}-2xy^{/}+2y=0$, about the point $x=0$. **(10)**

(b) Solve $Z(x+y)\frac{\partial Z}{\partial x}+Z(x-y)\frac{\partial Z}{\partial y}=(x^2+y^2)$. **(10)**

Q.No.3. **(a)** Classify the following equations: **(5)**

(i) $\frac{\partial^2 Z}{\partial x^2}+x^2\frac{\partial^2 Z}{\partial y^2}-\frac{1}{x}\frac{\partial Z}{\partial x}=0$

(ii) $x^2\frac{\partial^2 Z}{\partial x^2}+2xy\frac{\partial^2 Z}{\partial x\partial y}+y^2\frac{\partial^2 Z}{\partial y^2}=4x^2$

(b) Solve: $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, -1<x<1, t>0$ **(15)**

$u(-1, t) = u(1, t)$; $\frac{\partial u}{\partial x}(-1,t)=\frac{\partial u}{\partial x}(1,t)$ for $t > 0$

$u(x, o) = x+1$, $-1< x <1$.

SECTION-B

Q.No.4. **(a)** Highlight the difference between a vector and a tensor. What happens if we permute the subscripts of a tensor? **(5)**

(b) Transform $g^{ab}=\begin{pmatrix}1 & 0\\ 0 & 1/r^2\end{pmatrix}$ into Cartesian coordinates. **(15)**

Q.No.5. **(a)** Workout the Christoffel symbols for the metric tensor $g_{ab} = \begin{pmatrix} a^2 & 0 \\ 0 & a^2\sin^2\theta \end{pmatrix}$ **(10)**

(b) Workout the two dimensional metric tensor for the coordinates p and q given by $p = (xy)^{\frac{1}{3}}, \; q = \left(x^2 / y \right)^{1/3}$ **(10)**

SECTION-C

Q.No.6. **(a)** Solve the following system of equations by Jacobi iteration method: **(10)**

10x + y – 2z = 7.74
x + 12y +3z = 39.66
3x + 4y +15z = 54.8

(b) Solve $Sin x = 1 + x^3$ Using Newton-Raphson method. **(10)**

Q.No.7. **(a)** Find the root of $xe^x = 3$ by regular falsi method correct to three decimal places. **(10)**

(b) Evaluate $\int_0^{10} \frac{dx}{1+x^2}$ using **(5+5)** **(10)**

(i) Trapezoidal rule and
(ii) Simpson's rule.

Q.No.8. **(a)** Find the real root of the equation $Cos x = 3x - 1$ correct to seven decimal places by the iterative method. **(10)**

(b) Use Lagrange's interpolation formula to find the value of y when $x = 10$, if the values of x and y are given below: **(10)**

X	5	6	9	11
Y	12	13	14	16

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