Roll Number

FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2014

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE:(i) Candidate must write **Q.No.** in the **Answer Book** in accordance with **Q.No.** in the **Q.Paper.**

(ii) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B. ALL** questions carry **EQUAL** marks.

(iii) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.

(iv) Extra attempt of any question or any part of the attempted question will not be considered.

(v) **Use of Calculator is allowed.**

SECTION-A

Q. No. 1. **(a)** prove that $curl(¤\vec{F}) = (grad¤)$ x $\vec{F}$. If $\vec{F}$ is irrotational and $¤(x, y, z)$ is a scalar function. **(10)**

(b) Determine whether the line integral: **(10)**

$\int_c (2xyz^2 dx + (x^2z^2 + zCosyz)dy + (2x^2yz + yCosyz)dz$ is independent of the path of integration? If so, then compute it from (1,0,1) to $(0, \frac{®}{2}, 1)$.

Q. No. 2. **(a)** State and prove Stoke's Theorem. **(10)**

(b) Verify Stoke's Theorem for the function $F = x^2 i - xy\, j$ integrated round the square in the plane z=0 and bounded by the lines x = y = 0, x = y = a. **(10)**

Q. No. 3. **(a)** Three forces act perpendicularly to the sides of a triangle at their middle points and are proportional to the sides. Prove that they are in equilibrium. **(10)**

(b) Three forces P, Q, R act along the sides BC, CA, AB respectively of a triangle ABC. Prove that, if P Sec A + Q Sec B + R Sec C = 0, then the line of action of the resultant passes through the orthocentre of the triangle. **(10)**

Q. No. 4. **(a)** Find the centroid of the surface formed by the revolution of the cardioide $r = a(1 + Cos¯)$ about the initial line. **(10)**

(b) A uniform ladder rests with its upper end against a smooth vertical wall and its foot on rough horizontal ground. Show that the force of friction at the ground is $\frac{1}{2}W \tan¯$, where W is the weight of the ladder and ¯ is its inclination with the vertical. **(10)**

Q. No. 5. **(a)** Define briefly laws of friction give atleast one example of each law. **(10)**

(b) A uniform semi-circular wire hangs on a rough peg, the line joining its extremities making an angle of 45^o with the horizontal. If it is just on the point of slipping, find the coefficient of friction between the wire and the peg. **(10)**

APPLIED MATHEMATICS, PAPER-I

SECTION-B

Q. No. 6. **(a)** If a point P moves with a velocity v given by $v^2=n^2(ax^2+2bx+c)$, show that P executes a simple harmonic motion. Find the center, the amplitude and the time-period of the motion. **(10)**

(b) A particle P moves in a plane in such a way that at any time t its distance from a fixed point O is $r= at+bt^2$ and the line connecting O and P makes an angle $\bar{} = ct^{\frac{3}{2}}$ with a fixed line OA. Find the radial and transverse components of the velocity and acceleration of the particle at t =1. **(10)**

Q. No. 7. **(a)** A particle of mass m moves under the influence of the force $F = a(i\,Sin\,\mu\,t + j\,Cos\,\mu\,t)$. If the particles is initially at rest on the origin, prove that the work done upto time t is given by $\frac{a^2}{m\mu^2}(1-Cos\,\mu\,t)$, and that the instantaneous power applied is $\frac{a^2}{m\mu^2}Sin\,\mu\,t$. **(10)**

(b) A battleship is streaming ahead with speed V, and a gun is mounted on the battleship so as to point straight backwards, and is set an angle of elevation a, if v_o is the speed of projection relative to the gun, show that the range is $\frac{2v_o}{g}Sin\sim(v_oCos\sim -V)$. Also prove that the angle of elevation for maximum range is $arcCos\left(\frac{V-\sqrt{V^2-8v_0^2}}{4v_0}\right)$. **(10)**

Q. No. 8. **(a)** Show that the law of force towards the pole, of a particle describing the curve $r^n = a^n Cos\, n\bar{}$ is given by $f = \frac{(n+1)h^2a^{2n}}{r^{2n+3}}$. **(10)**

(b) A bar 2 ft. long of mass 10 Ib., lies on a smooth horizontal table. It is struck horizontally at a distance of 6 inches from one end, the blow being perpendicular to the bar. The magnitude of the blow is such that it would impart a velocity of 3 ft./sec. to a mass of 2 Ib. Find the velocities of the ends of the bar just after it is struck. **(10)**

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APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS: 100**

NOTE:(i) Candidate must write **Q.No**. in the **Answer Book** in accordance with **Q.No**. in the **Q.Paper.**
(ii) Attempt FIVE questions in all by selecting **TWO** questions from **SECTION-A, ONE** question from **SECTION-B** and **TWO** questions from **SECTION-C. ALL** questions carry **EQUAL** marks.
(iii) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(iv) Extra attempt of any question or any part of the attempted question will not be considered.
(v) **Use of Calculator is allowed.**

SECTION-A

Q. No. 1. **(a)** Solve the initial-value problem **(10)**

$$\frac{dy}{dx} = \frac{1}{x + y^2}; \quad y(-2) = 0 .$$

(b) Initially there were 100 milligrams of a radioactive substance present. After 6 hours the mass decreased by 3%. If the rate of decay is proportional to the amount of the substance present at any time, find the amount remaining after 24 hours. **(10)**

Q. No. 2. **(a)** Solve $(x^2 + 1)y'' + xy' - y = 0$. **(10)**

(b) Obtain the partial differential equation by elimination of arbitrary functions, **(10)**
$a \sin x + b \cos y = z$ (take z as dependent variable).

Q. No. 3. **(a)** Solve the partial differential equation $u_{xx} + u_{yy} = u_t$, **(10)**

subject to the conditions
$$u(0, y, t) = u(a, y, t) = 0$$
$$u(x, 0, t) = u(x, a, t) = 0$$
and the initial condition, $u(x, y, 0) = ¤(x, y)$.

(b) Solve $r + (a + b)s + abt = xy$ by Monge's method. **(10)**

SECTION-B

Q. No. 4. **(a)** Prove that if $A_i, B_j, and\ C_k$ are three first order tensors, then their product $A_i B_j C_k \ (i, j, k = 1, 2, 3)$ is a tensor of order 3, while $A_i B_j C_k (i, j = 1, 2, 3)$ form a first order tensor. **(10)**

(b) If $A_{i_1 i_2 i_3 \cdots i_n}$ is a tensor of order *n*, then its partial derivative with respect to x_p that is $\frac{\partial}{\partial x_p} A_{i_1 i_2 i_3 \cdots i_n}$ is also a tensor of order *n+1*. **(10)**

Q. No. 5. **(a)** Show that the transformation $\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -3 & -6 & -2 \\ -2 & 3 & -6 \\ 6 & -2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is orthogonal and right-handed. **(10)**

A second order tensor A_{ij} is defined in the system $Ox_1x_2x_3$ by

$A_{ij} = x_i x_j \quad i, j = 1,2,3.$ Evaluate its components at the point P where

$x_1 = 0,\ x_2 = x_3 = 1.$ Also evaluate the component A_{11}' of the tensor at P.

(b) The Christofell symbols of the second kind denoted by $\begin{Bmatrix} m \\ ij \end{Bmatrix}$ are defined **(10)**

$$\begin{Bmatrix} m \\ ij \end{Bmatrix} = g^{mk}\left[ij,k\right] \quad (i, j, k = 1,2,...n).$$

Prove that (i) $\begin{Bmatrix} m \\ ij \end{Bmatrix} = \begin{Bmatrix} m \\ ji \end{Bmatrix}$, (ii) $[ij,k] = g_{mk}\begin{Bmatrix} m \\ ij \end{Bmatrix}$,

(iii) $\dfrac{\partial g^{ij}}{\partial x^k} = -g^{im}\begin{Bmatrix} i \\ km \end{Bmatrix} - g^{jm}\begin{Bmatrix} i \\ km \end{Bmatrix}$.

SECTION-C

Q. No. 6. **(a)** Apply Newton-Raphson's method to determine a root of the equation $f(x) = \cos x - xe^x = 0$ such that $\left|f(x^*)\right| < 10^{-8}$, where x^* is the approximation to the root. **(10)**

(b) Consider the system of the equations **(10)**

$$2x_1 - x_2 + 0x_3 = 7$$
$$-x_1 + 2x_2 - x_3 = 1,$$
$$0x_1 - x_2 + 2x_3 = 1$$

Solve the system by using Gauss-Seidel iterative method and perform three iterations.

Q. No. 7. **(a)** Use the trapezoidal and Simpson's rules to estimate the integral **(10)**

$$\int_1^3 f(x)dx = \int_1^3 (x^3 - 2x^2 + 7x - 5)dx\ .$$

(b) Find the approximate root of the equation $f(x) = 2x^3 + x - 2 = 0$. **(10)**

Q. No. 8. **(a)** Find a 5th degree polynomial which passes through the 6 points given below. **(10)**

x	1.0	2.0	4.0	5.0	7.0	8.0
$f(x)$	-9	-41	-189	-173	9	523

(b) Determine the optimal solution graphically to the linear programming problem, **(10)**

$$\begin{aligned} &\textit{Minimize} \quad z = 3x_1 + 6x_2 \\ &\textit{subject to} \quad 4x_1 + x_2 \geq 20 \\ &\qquad x_1 + x_2 \leq 20 \\ &\qquad x_1 + x_2 \geq 10 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

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