FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2015

Roll Number

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **ONLY FIVE** questions in all, by selecting **THREE** questions from **SECTION-I** and **TWO** questions from **SECTION-II. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-I

Q. No. 1 **(a)** Prove that $(\vec{A}+\vec{B})\cdot(\vec{B}+\vec{C})\times(\vec{C}+\vec{A}) = 2[\vec{A}\cdot(\vec{B}\times\vec{C})]$. **(10)**

(b) If $\vec{A} = (x-3y)\hat{i} + (y-2x)\hat{j}$, evaluate $\oint_c \vec{A}.d\vec{r}$ where c is an ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ in the xy- plane traversed in the positive direction. **(10)**

Q. No. 2 **(a)** Determine the expression for divergence in orthogonal curvilinear coordinates. **(10)**
(b) Determine the unit vectors in spherical coordinate system. **(10)**

Q. No. 3 **(a)** A particle moves from rest at a distance "*a*" from a fixed point *O* where the acceleration at distance *x* is $«x^{-\frac{5}{3}}$. Show that the time taken to arrive at *O* is given by an equation of the form $t = A\frac{a^{\frac{4}{3}}}{\sqrt{«}}$, where *A* is a number. **(10)**

(b) Three forces *P*, *Q*, *R* acting at a point, are in equilibrium, and the angle between *P* and *Q* is double of the angle between *P* and *R*. Prove that $R^2 = Q(Q-P)$. **(10)**

Q. No. 4 **(a)** *AB* and *AC* are similar uniform rods, of length *a*, smoothly joined at *A*.*BD* is a weightless bar, of length *b*, smoothly joined at *B*, and fastened at *D* to a smooth ring sliding on *AC*. The system is hung on a small smooth pin at *A*. Show that the rod *AC* makes with the vertical an angle $\tan^{-1}\frac{b}{a+\sqrt{a^2-b^2}}$. **(10)**

(b) Find the centroid of the arc of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ lying in the first quadrant. **(10)**

Q. No. 5 **(a)** A hemispherical shell rests on a rough inclined plane whose angle of friction is ª. Show that the inclination of the plane base to the horizontal cannot be greater than $\sin^{-1}(2\sin ª)$. **(10)**

(b) A regular octahedron formed of twelve equal rods, each of weight *w*, freely jointed together is suspended from one corner. Show that the thrust in each horizontal rod is $\frac{3}{2}\sqrt{2}w$. **(10)**

APPLIED MATHEMATICS, PAPER-I

SECTION-II

Q. No. 6 **(a)** A particle is moving with uniform speed v along the curve $x^2 y = a(x^2 + \frac{a^2}{\sqrt{5}})$. **(10)**

Show that its acceleration has the maximum value $\frac{10v^2}{9a}$.

(b) Discuss the motion of a particle moving in a straight line if it starts from rest at a distance a from a point O and moves with an acceleration equal to « times its distance from O. **(10)**

Q. No. 7 **(a)** Prove that the force field **(10)**

$F = (y^2 - 2xyz^3)i + (3 + 2xy - x^2y^3)j + (6z^3 - 3x^2yz^2)k$

is conservative, and determine its potential.

(b) The components of velocity along and perpendicular to the radius vector form a fixed origin are respectively ªr^2 and «¯^{2}. **(10)**

Find the polar equation of the path of the particle in terms of r and ¯ .

Q. No. 8 **(a)** A particle is projected horizontally from the lowest point of a rough sphere of radius a. After describing an arc less than a quadrant, it returns and comes to rest at the lowest point. Show that the initial speed must be (sin~)$\sqrt{\frac{2ag(1+«^2)}{(1-2«^2)}}$, **(10)**

Where « is the coefficient of friction and a~ is the arc through which the particle moves.

(b) The law of force is Mu^2 and a particle is projected from or apse at distance a. Find the orbit when the velocity of the projection is $\frac{\sqrt{M}}{a^2}$. **(10)**

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APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE:(i) Attempt **FIVE** questions in all by selecting **TWO** questions from **SECTION-A, ONE** question from **SECTION-B** and **TWO** questions from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) Candidate must write **Q.No**. in the **Answer Book** in accordance with **Q.No**. in the **Q.Paper.**
(iii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iv) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(v) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(vi) Extra attempt of any question or any part of the attempted question will not be considered.
(vii) **Use of Calculator is allowed.**

SECTION-A

Q. No. 1. **(a)** Solve the initial value problem. **(10)**

$$\frac{dy}{dx}+\frac{y}{2x}=\frac{x}{y^3},\ y(1)=2$$

(b) Solve $y''-4y'+4y=e^{2x}$ **(10)**

Q. No. 2. Solve the following equations: **(10)**

(a) $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$

(b) $\frac{d^3y}{dx^3}+\frac{dy}{dx}=\cos ecx$ **(10)**

Q. No. 3. **(a)** Classify the following: **(5 each)** **(10)**

(i) $x^2U_{xx}+(a^2-y^2)U_{yy}=0 \quad , \quad -\infty<x<\infty,\ -a<y<a$

(ii) $U_{xx}-6U_{xy}+9U_{yy}+3y=0$

(b) Solve **(10)**

$$\frac{\partial^2 u}{\partial^2 t}=\frac{\partial^2 u}{\partial^2 x} \qquad 0<x<5$$
$$u(0,t)=u(5,t)=0$$
$$u(x,0)=x^2(x-5)$$
$$u_t(x,0)=0$$

SECTION-B

Q. No. 4. **(a)** Prove that if A_i and B_j are two first order tensors, then their product $A_iB_j\,(i,j=1,2,3)$ is a second order tensor. **(7)**

(b) If ¤(x_1,x_2,x_3) is a scalar point function then $\frac{\partial ¤}{\partial x_i}$ are the components of a first order tensor. **(7)**

(c) Find the invariant of the following second order tensor **(6)**

$$\begin{bmatrix} 2 & 4 & -1 \\ 6 & -7 & 10 \\ 3 & -4 & 6 \end{bmatrix}$$

APPLIED MATHEMATICS, PAPER-II

Q. No. 5. **(a)** Verify that the transformation **(7)**

$$x_1' = \frac{1}{15}(5x_1 - 14x_2 + 2x_3)$$

$$x_2' = -\frac{1}{3}(2x_1 + x_2 + 2x_3)$$

$$x_3' = \frac{1}{15}(10x_1 + 2x_2 - 11x_3)$$

Is orthogonal and right handed. A vector field $\vec{A}$ is defined in the system

$Ox_1x_2x_3$ by $A_1 = x_1^2, A_2 = x_2^2, A_3 = x_3^2$

Evaluate the components A_j' of the vector field in the new system $Ox_1'x_2'x_3'$.

(b) Prove that any second order tensor A_{ij} can be written as the sum of a deviator and an isotropic tensor. **(7)**

(c) If $a_{ij} = a_{ji}$ are constants. Calculate. **(6)**

$$\frac{\partial^2}{\partial X_k \partial X_m}(a_{ij} X_i X_j)$$

SECTION-C

Q. No. 6. **(a)** Find the real root of the equation by using Newton – Raphson's method. **(10)**

$3x - \cos x - 1 = 0$

(b) Solve the following system of equations by Gauss-Seidel method. Take initial approximation as $x_1 = 0, \quad x_2 = 0, \quad x_3 = 0$. Perform 3 Iterations. **(10)**

$20x_1 + x_2 - 2x_3 = 17$

$3x_1 + 20x_2 - x_3 = -18$

$2x_1 - 3x_2 + 20x_3 = 25$

Q. No. 7. **(a)** Find the real root of the equation $x^3 - 4x - 9 = 0$ by Regular falsi method. Take the interval of the root as (2,3) and perform 4 iterations. . **(10)**

(b) Find a polynomial which possess through the following points: **(10)**

x:	-1	0	1	2
$f(x)$:	2	1	2	5

Q. No. 8. **(a)** Use the langrage's Interpolation formula to find the value *f*(12) if the values of *x* and *f(x)* are given below **(10)**

x:	5	7	11	13
f(x)	150	392	1452	2366

(10)

(b) Evaluate $\int_0^3 x\sqrt{1+x^2}\,dx$ using $\frac{1}{3}$ Simpson's rule and trapezoidal rule for $n = 6$

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