Q. No. 1. 
(a) Prove that  \( \nabla \left[ \frac{f(r)}{r} \right] = 2 \frac{f(r)}{r} + f'(r) \) \( \quad (10) \)

(b) Verify Stokes’ theorem for  \( \mathbf{A} = (2x - y) \mathbf{i} - yz \mathbf{j} - y^2 z \mathbf{k} \), where  \( S \) is the upper half surface of the sphere  \( x^2 + y^2 + z^2 = 1 \) and  \( C \) is its boundary. \( \quad (10) \)

Q. No. 2. 
(a) Forces  \( P, Q, R \) act at a point parallel to the sides of a triangle  \( ABC \) taken in the same order. Show that the magnitude of the resultant force is \( \sqrt{P^2 + Q^2 + R^2 - 2QR \cos A - 2RP \cos B - 2PQ \cos C} \) \( \quad (10) \)

(b) Find the distance from the cusp of the centroid of the region bounded by the cardioid  \( r = a (1 + \cos \theta) \). \( \quad (10) \)

Q. No. 3. 
(a) A particle describes simple harmonic motion in such a way that its velocity and acceleration at a point  \( P \) are  \( u \) and  \( f \) respectively and the corresponding quantities at another point  \( Q \) are  \( v \) and  \( g \). Find the distance  \( PQ \). \( \quad (10) \)

(b) Derive the radial and transverse components of velocity and acceleration of a particle. \( \quad (10) \)

Q. No. 4. 
Solve the following differential equations:

(a) \( \frac{d}{dx} \left[ \frac{y}{x} \right] + y = x^3 y^4 \) \( \quad (10) \)

(b) \( (D^2 - 5D + 6) y = x^3 e^{2x} \) \( \quad (10) \)

Q. No. 5. 
(a) Solve the differential equation using the method of variation of parameters \( \frac{d^2 y}{dx^2} + y = \tan x \), \( -\frac{\pi}{2} < x < \frac{\pi}{2} \) \( \quad (10) \)

(b) Solve the Euler – Cauchy differential equation \( x^2 y'' - 3x y' + 4y = x^2 \ln x \). \( \quad (10) \)

Q. No. 6. 
(a) Find the Fourier series of the following function:
\[ f(x) = \begin{cases} -x & \text{if } -\pi < x < 0 \\ 0 & \text{if } 0 < x < \pi \end{cases} \]
\( \quad (10) \)

(b) Solve the initial - boundary value problem: \( \quad (10) \)
Q. No. 7.  (a) Apply Newton – Raphson method to find the smaller positive root of the equation
\[ x^2 - 4x + 2 = 0 \]
(b) Solve the following system of equations by Gauss – Seidel iterative method by
taking the initial approximation as \( x_1 = 0, \ x_2 = 0, \ x_3 = 0 \):
\[
\begin{align*}
5x_1 + x_2 - x_3 &= 4 \\
x_1 + 4x_2 + 2x_3 &= 15 \\
x_1 - 2x_2 + 5x_3 &= 12
\end{align*}
\]

Q. No. 8.  (a) Approximate \( \int_0^1 \frac{dx}{1 + x^2} \) using
(i) Trapezoidal rule with \( n = 4 \)  (ii) Simpson’s rule with \( n = 4 \)
Also compare the results with the exact value of the integral.
(b) Apply the improved Euler method to solve the initial – value problem:
\[ y' = x + y, \quad y(0) = 0 \]
by choosing \( h = 0.2 \) and computing \( y_1, \ldots, y_5 \).