FEDERAL PUBLIC SERVICE COMMISSION

Roll Number

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2011

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE: **(i)** Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION – A** and **TWO** questions from **SECTION – B.** All questions carry equal marks.
(ii) **Use of Scientific Calculator is allowed.**
(iii) **Extra attempt of any question or any part of the attempted question will not be considered.**

SECTION - A

Q.1. (a) Prove that both the order and index of a subgroup of a finite group divide the order of the group. **(10)**

(b) Define cyclic group. Also prove that every cyclic group is abelian. **(05)**

(c) Define order of a permutation in S_n. Find the order of $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ **(05)**

Q.2. (a) Let ϕ be a homomorphism of a group G onto another group H with Kernel K. Prove that G/K is isomorphic to H. **(10)**

(b) Show that the vectors (3, 0, -3), (-1, 1, 2), (4, 2, -2) and (2, 1, 1) are linearly dependent over R. **(10)**

Q.3. (a) Define the dimension of a vector space V over a field F. Also prove that all basis of a finite dimensional vector space contain the same number of elements. **(10)**

(b) A linear transformation $T: U \rightarrow V$ is one –to-one iff N(T) ={0}. **(10)**

Q.4. (a) Examine the following system for a non-trivial solution: **(10)**

$$\begin{aligned} x_1 - x_2 + 2x_3 \quad + x_4 &= 0 \\ 3x_1 + 2x_2 \quad\quad + x_4 &= 0 \\ 4x_1 + x_2 + 2x_3 \quad + 2x_4 &= 0 \end{aligned}$$

(b) Show that $\overline{Z}_3 = \{\overline{0}, \overline{1}, \overline{2}\}$ form finite field with addition and multiplication of residue classes modulo P. **(10)**

Q.5. (a) Let V be a vector space of n – square matrices over a field R. Let U and W be the subspaces of symmetric and anti symmetric matrices respectively. Then show that V = U O W. **(10)**

(b) Let A and B be matrices of order 6 such that det (AB^2) = 72 and det (A^2B^2) = 144. Find det (A) and det (AB^6) **(10)**

SECTION – B

Q.6. (a) Sketch the curve $r^2 = a^2 \cos 2\theta, \quad a > 0.$ **(10)**

(b) Find the tangent and the normal to the circle x =a cosθ, y = a sin θ at the point P (a cos α, a sin α). **(10)**

Q.7. (a) Find the Pedal equation of the parabola $y^2 = 4a(x+a)$ **(10)**

(b) Find the equations for a straight line passing through the points $P_1(x_1, y_1, z_1), P_2(x_2, y_2, z_2)$. Find the co-ordinates of the point where this line cuts the yz-plane. **(10)**

Q.8. (a) Determine the curvature of the cycloid x = a (t – sin t), y = a(1 – cos t) at the point (x,y). **(10)**

(b) Find the equation of the plane which passes through the point (3, 4, 5) has an x – intercept equal to -5 and is perpendicular to the plane 2x + 3y – z = 8. **(10)**

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COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2011

Roll Number

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS | **MAXIMUM MARKS: 100**

NOTE: **(i)** Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION – A** and **TWO** questions from **SECTION – B.** All questions carry equal marks.

(ii) **Use of Scientific Calculator is allowed.**

(iii) **Extra attempt of any question or any part of the attempted question will not be considered.**

SECTION - A

Q.1. (a) Prove that every non-empty set of real numbers that has an upper bound also has an supremum in R. **(10)**

(b) If $x \in \boldsymbol{R}$, set of real numbers, then there exists $\mathbf{n} \in \mathbf{N}$ such that $\mathbf{x} < \mathbf{n}$. **(10)**

Q.2. (a) Define continuity of a function at a point and also prove that if **f** and **g** be functions on **A** to $\boldsymbol{R}$, where $\mathbf{A} \subseteq \mathrm{R}$ then **f + g** and f g are continuous at **C**. **(10)**

(b) If $\mathbf{f : I \to R}$ is differentiable at $\mathbf{C} \in \mathbf{I}$, then f is continuous at **C**. **(10)**

Q.3. (a) Evaluate $\int_1^5 \frac{dx}{\sqrt[3]{x-2}}$. **(08)**

(b) (i) Define Complete metric space. **(04)**

(ii) Prove that a sequence of real numbers is convergent iff it is a Cauchy sequence. This theorem is not in metric space, for justification give one example. **(08)**

Q.4. (a) Let **(x, d)** be a matric space and **A** a subset of **X**. Then prove that

(i) Interior A° of A is an open subset of **X**. **(05)**

(ii) A° is the largest subset of **X** contained in **A**. **(05)**

(b) State and prove Mean value theorem. **(10)**

Q.5. (a) If $\sum a_n$ converges absolutely then $\sum a_n$ converges. **(10)**

(b) Find the area enclosed by the parabola $y^2 + 16x - 71 = 0$ and the line $4x + y + 7 = 0$ **(10)**

SECTION – B

Q.6. (a) Let $Z = (\cos\theta + i\,Sin\theta)$. Then prove that $Z^n = Cos\,n\theta\ i\,Sin\,n\theta$ for all **n**. **(10)**

(b) Using De Moivre's Theorem evaluate $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^6$. **(10)**

Q.7. (a) Expand $\mathbf{f(x) = x^2, 0 < x < 2\pi}$ in a Fourier series if period is 2π. **(10)**

(b) If f(z) is analytic inside a circle C with centre at a, then for all Z inside C **(10)**

$$f(z) = f(a) + f'(a)(z-a) + \frac{f''(a)}{2!}(z-a)^2 + \ldots$$

Q.8. (a) Evaluate the integral by using Cauchy integral Formula **(10)**

$$\int_c \frac{(4-3z)dz}{z(z-1)(z-2)} \quad \text{where C is a circle } |z| = 3/2.$$

(b) Prove that **(10)**

$$\int_0^{2\pi} \frac{d\theta}{1 - 2pCos\theta - p^2} = \frac{2\pi}{1-p^2}.$$

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