FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION – 2026 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	PART-I (MCQs) MAXIMUM MARKS: 20
PART-I (MCQs): MAXIMUM 30 MINUTES	PART-II MAXIMUM MARKS: 80

NOTE: (i) First attempt PART-I (MCQs) on separate OMR Answer Sheet which shall be taken back after 30 minutes.
(ii) Overwriting/cutting of the options/answers will not be given credit.
(iii) There is no negative marking. All MCQs must be attempted.

SECTION-A

Q. No. 1

(a) State the Lagrange's Theorem, and find all the subgroups of the cyclic group of order 24. Show that any group of prime order has no non-trivial subgroups. (10)

(b) Let G be an Abelian group and H is a subset of G consisting of those elements of G which are of the second order then H is a subgroup of G. Let

$$G = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in R, ad - bc \neq 0 \right\}$$

be the group of all matrices under matrix multiplication. Show that

$$H = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in R, ad \neq 0 \right\}$$

is a subgroup of G and

$$K = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in R \right\}$$

is a subgroup of H. (10)

Q. No. 2

(a) Consider the group Z_6 under addition modulo 6. Let

$$\psi(x) = 5x(\mathrm{mod}6).$$

Verify that it is an automorphism. Define Kernel and Image with at least one example of each. Consider the homomorphism

$$\psi: Z_4 \to Z_2,$$

where

$$\psi(x) = x(\mathrm{mod} 2).$$

Find $Ker(\psi)$? Let $\phi: G \rightarrow H$be a group homomorphism, and let $g \in G$. Then for any integer n, we have

$$\phi(g^n) = \phi(g)^n.$$

(b) A computer chip manufacturer decides to spend 6×10^5rupees on radio, magazine, and TV advertisement. If he spends as much on TV advertisement as on magazine and radio together and the amount spent on magazine and TV combined equals five times that spent on radio. What is the amount to be spent on each type of advertisement? Use Gauss Jordan Method. (10)

Q. No. 3

(a) Find Echelon form and Rank of the following matrix by successively reducing it to its row/column sub-matrices.

$$\begin{bmatrix} 1 & -2 & 1 & 0 & 5 \\ -1 & 0 & 1 & -2 & 2 \\ 2 & -3 & 0 & 2 & 3 \end{bmatrix}$$

(b) Define linearly independent and linearly dependent vectors. Are the vectors

$$(1,-2,4,1), (2,1,0,-3) \text{and} (1,-6,1,4)$$

in R^4linearly independent or linearly dependent? (10)

SECTION-B

Q. No. 4

(a) Consider two planes as follows:

$$P_1: 2x - y - 2z + 5 = 0,$$
$$P_2: 4x - 2y - 4z + 15 = 0.$$

Are the planes parallel? If no, what is angle between them? (10)

(b) Derive the equation of sphere in standard form. Find the equation of sphere through the circle:

$$x^2 + y^2 + z^2 = 9, 2x + 4y + 5z = 6$$

and touching the xz-plane.

Q. No. 5.

(a) Find cylindrical coordinates of the point with rectangular coordinates $(2\sqrt{3}, 2, -2)$. Express the equation

$$(x+y)^2 - z^2 + 4 = 0$$

in cylindrical and spherical coordinates.

(b) Use Maclaurin Series to prove that

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

hence find the value of π.

Q. No. 6.

(a) Evaluate the following limits:

$$\text{(i)}\lim_{x \to 0} \left[\frac{1}{\sin\ 3x} - \frac{1}{3x}\right]$$

$$\text{(ii)}\lim_{x \to 0} \frac{x - \sin\ x}{x^3}$$

(b) What is the area A, of the region bounded by the curves

$$y = x^2$$

and

$$y = \sqrt{x}\,?$$

SECTION-C

Q. No. 7.

(a) State De-Moivre's Theorem. If

$$x + \frac{1}{x} = 2\cos\ \theta, y + \frac{1}{y} = 2\cos\ \varphi, z + \frac{1}{z} = 2\cos\ \phi,$$

prove that

$$xyz + (xyz)^{-1} = 2\cos\,(\theta + \varphi + \phi).$$

(b) State Green's Theorem and verify it for

$$\int_C (2xy - x^2)\, dx + (x + y^2)\, dy,$$

where $C$is the curve bounded by

$$y = x^2$$

and

$$x = y^2.$$

Q. No. 8.

(a) Use the concept of Path Integrals to prove following:

(i) $\int_{|z|=1} \frac{1}{z}\, dz = 2\pi i$

(ii) $\int_{|z|=1} z\, dz = 0$

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(b) State Residue Theorem and use it to prove the following:

$$\oint_C \frac{e^{-z}}{(z+2)^5}\, dz = \frac{1}{12}\pi i e^2$$

where $C$is the circle